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Entropy and Black-Hole Evaporation

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The aim of the present contribution is to exemplify, in a phenomenological way, a nontraditional cosmology which includes particle production, and, therefore, cosmological entropy production. Before making these considerations explicit, let us recall the previous results obtained in the framework of semiclassical gravity [see Nardone (1989) and references therein]. The general question referring to the proposal presented by Nardone can be put in the following way: in the beginning were two empty infinite reservoirs, the reservoir of possibles, which constitutes the quantum vacuum, and the reservoir of energy, which constitutes the curvature of space-time. In fact, the more space-time is curved, the more the energy which characterizes it appears to be negative. The question is then: can the physically conceivable mechanism of the creation of matter *ex nihilo* be actually put into action? Can the two reservoirs be put into communication in a way that brings virtual particles into existence with an energy supplied by curvature? If the reply to these questions is positive, if some virtual particles, vacuum fluctuations, which, like everything, are sources of gravitation and retroactively feel its effect, could provoke a curvature such that the corresponding gravitational energy be sufficient to realize them, we should have, thereby, the origin of a history, in which gravitation, through its self-amplifying properties, constitutes a spring. A chain process would, in fact, be set up: the realization of virtual particles would accentuate the curvature of space-time, which, in turn, would create new particles.... The reply to the question is positive: the empty quantum universe is unstable with respect to the occurrence of particles of mass above a definite threshold, corresponding to about 50 "Planck masses" (the mass built out of the only three universal constants, Planck's constant, the velocity of light, and the gravitational constant, and with a value of some 10^{-59}). It consequently

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turns out that the primordial cosmological actors, from the quasimacroscopic mass which they must have, probably belong to that most fascinating class of entities ever constructed by physicists: black holes.

Black holes conjure up in the popular imagination the end of all history, the collapse of matter, without recovery, into the gravitational trap which it generates. However, black holes, like all objects in contemporary physics, are hybrid beings. Gravitation defines them as a trap and a singularity, but quantum mechanics obliges them to evaporate, attributes to them a lifetime, a temperature (that of the particles and radiation which the black hole releases), and hence an entropy. It is in this sense that black holes can feature not at the end, but at the beginning of cosmological history, and that the products of their evaporation become the constitutents, light and particles, of our universe.

A black hole of 50 times the Planck mass will live for about 10^{-37} sec. This would, therefore, be the duration of the "birth" of the universe, of the self-catalytic transformation of gravitational energy into matter. The entropy of the black holes, deemed to be created during the "birth" of the universe, is precisely that which characterizes the universe we observe!

We present the simplest possible phenomenological model wherein the irreversible creation phenomenon is expressed in terms of the Hubble function H as follows:

$$\frac{1}{R^3}\frac{d(nR^3)}{d\tau}=\alpha H^2 \geq 0 \qquad \text{with} \quad \alpha \geq 0 \tag{1}$$

To complete the model, we assume the simple relation $\rho = Mn$; hence $p=0$.

For $\alpha = 0$ we recover as the solution for the spatially flat Einstein equation

$$\kappa\rho = 3H^2$$

which is the usual RW description with its typical big-bang singularity. However, for $\alpha \neq 0$, we obtain

$$p=0$$

$$\rho=\frac{3}{\kappa}H^2$$

$$\frac{1}{nR^3}\frac{d(nR^3)}{d\tau}=\frac{\alpha\kappa M}{3}\geq 0 \tag{2}$$

This leads to

$$N(\tau)=N_0\, e^{\alpha\kappa M\tau/3}$$

and

$$R(\tau) = [1 + C(e^{\alpha\kappa M\tau/6} - 1)]^{2/3}$$

where

$$C = \frac{9}{\kappa M\alpha}\left(\frac{\kappa M n_0}{3}\right)^{1/2}$$

The universe emerges without singularity ($R \neq 0$) at $\tau = 0$, with a particle density n_0 describing the initial Minkowskian fluctuation. It therefore follows that the presence of dissipative particle creation ($\alpha \neq 0$) leads to the disappearance of the big-bang singularity. In other words, this singularity is *structurally unstable* with respect to irreversible particle creation. Hence, such a cosmology starts from an instability ($n_0 \neq 0$), and not from a singularity.

After a characteristic time

$$\tau_c = \frac{6}{\alpha\kappa M} \tag{3}$$

the universe reaches a de Sitter regime characterized by

$$\begin{aligned} R_d(\tau) &= C^{2/3}\, e^{\alpha\kappa M\tau/9} = C^{2/3}\, e^{2\tau/3\tau_c} \\ H_d &= \frac{\alpha\kappa M}{9} = \frac{2}{3\tau_c} \\ n_d &= \frac{\kappa M}{27}\alpha^2 \end{aligned} \tag{4}$$

It is remarkable that all the de Sitter physical quantities, such as H_d, n_d, and ρ_d, are independent of C. This cosmological state therefore appears to be an attractor independent of the initial fluctuation. The de Sitter stage survives during the decay time τ_d of its constituents and then connects continuously (up to the first derivatives) to a usual (adiabatic) matter-radiation RW universe characterized by a matter-energy density ρ_b and radiation energy density ρ_γ related to the RW function by

$$\kappa\rho_b = \frac{3a}{R^3}, \qquad \kappa\rho_\gamma = \frac{3b}{R^4}, \qquad \rho_\gamma = \frac{\pi^2}{15}T^4 \tag{5}$$

a and b are constants related to the total number N_b of baryons and photons N_γ in a volume R^3, and T is the blackbody radiation temperature. The connection at the decay time τ_d between the de Sitter and the matter-radiation regimes fixes the constants a and b:

$$\begin{aligned} a &= 2H_d^2 C^2\, e^{2H_d\tau_d} \\ b &= H_d^2 C^{8/3}\, e^{4H_d\tau_d} \end{aligned} \tag{6}$$

This implies that the (constant) specific entropy S per proton is

$$S = \frac{n_\gamma}{n_b} = \frac{\zeta(3)}{3\pi^2}\left(\frac{45}{\pi^2}\right)^{3/4} \kappa^{1/4} m_b \left(\frac{3\tau_d}{2}\right)^{1/2} e^{2\tau_d/3\tau_c} \tag{7}$$

where m_b stands for the proton mass. Similarly, the value of the adiabatic invariant is $\rho_\gamma / T\rho_b$:

$$\frac{\rho_\gamma}{T\rho_b} = \frac{b^{3/4}}{a}\left(\frac{\pi^2 \kappa}{45}\right)^{1/4} \tag{8}$$

Hence these values (7), (8) are entirely fixed by the knowledge of the two characteristic times τ_c and τ_d [see (4)]. In our previous work the subject was treated quantum mechanically. In this context, both quantities τ_c and τ_d were expressed in terms of one single parameter, namely the mass M of the produced particles. These values are

$$\tau_d = \frac{640}{81\pi}\kappa^2 M^3 = \frac{640}{81\pi}\frac{M^3}{M_p^4} \simeq 2.5\left(\frac{M}{M_p}\right)^3 \tau_p$$

and

$$\tau_c = \frac{2}{3H_d} = \left(\frac{20}{\pi^2}\right)^{1/2}\frac{M^2}{M_p^3} \simeq 1.42\left(\frac{M}{M_p}\right)^2 \tau_p$$

where M_p and τ_p are the Planck mass and the Planck time, respectively.

Although highly sensitive (exponentially) to the value of the mass M, it is quite remarkable that correct observe values for S, namely $10^8 \leq S \leq 10^{10}$, are then obtained for values of the mass M very close to the quantum mechanically produced one ($53.3 M_p$), for example,[2]

$$M/M_p = 40 \rightarrow S = 8.46 \times 10^2$$

$$M/M_p = 50 \rightarrow S = 1.38 \times 10^8$$

$$M/M_p = 53.3 \rightarrow S = 7.17 \times 10^9$$

$$M/M_p = 60 \rightarrow S = 2.16 \times 10^{13}$$

This fact provides an unexpected link between the microscopic and macroscopic approaches.

[2]When one takes into account the N helicity states associated with the massless particles present in the cosmological medium, the critical mass M is increased by a factor of $O(\sqrt{N})$. Moreover, this critical mass can be shown to originate in the Minkowskian vacuum in a process very similar to the inverse Hawking black-hole evaporation process. These two points will be reported in detail in a forthcoming publication.

Moreover, the present blackbody temperature is deduced from the continuity requirements to be

$$T_p = \left(\frac{45}{\pi^2}\kappa^{-1}\right)^{1/4} \frac{b^{1/4}}{a^{1/3}} H_p^{2/3} \tag{9}$$

$$T_p(K) \simeq 2.82 \times 10^{-9} \left(\frac{H_p}{75 \text{ km/sec/Mpc}}\right)^{2/3} \left(\frac{M}{M_p}\right)^{1/3} e^{0.3926 M/M_p} \tag{10}$$

where H_p is the presently observed value for the Hubble function:

$$50 \leq H_p(\text{km/sec/Mpc}) \leq 100$$

Values well in the range of the observed blackbody radiation temperture (2.7 K) are also obtained with the same values of the mass M:

$$M/M_p = 50 \to T_p = 3.49 \left(\frac{H_p}{75 \text{ km/sec/Mpc}}\right)^{2/3} \text{K}$$

In conclusion, a main feature of our model is that it takes into account the second law of thermodynamics at the cosmological scale, from the very beginning. Indeed, the energy transfer from space-time curvature to matter appears to be an irreversible process leading to a burst of entropy associated with the creation of matter. It follows therefore that the distinction between space-time and matter is provided by entropy creation. The question of the cosmological arrow of time is far from being a simple one, as both expansion and contraction can occur reversibly, as in traditional cosmology. However, the conditon for the creation of matter from space-time involves the sign of the Hubble function. As mentioned above, in the case of the de Sitter universe, expansion only is thermodynamically possible. As, according to the nontraditional cosmology presented here, the universe always develops through a de Sitter stage, there is indeed a direct relation between the existence of cosmological entropy and the expansion of the universe.

REFERENCE

Nardone, P. (1989). *International Journal of Theoretical Physics*, this issue.